THERMAL STATE OF A POROUS PLATE COOLED BY STRONG INJECTION DURING RADIATIVE – CONVECTIVE HEATING

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Experiments and calculations have revealed the characteristics of internal cooling of a porous plate by strong injection during radiative–convective heating.

In many power devices operating on forced process modes (combustion chambers, electric-arc gas preheaters, aircraft and rocket engines, heat exchangers, MGD devices, etc.) radiation is increasingly dominating the thermal effects on the wall surfaces. When porous injection is applied as a method of thermal protection, there often occurs a situation where mostly radiative heat is removed from the porous wall by internal cooling. Such situations are encountered mainly during the injection of a transparent gas which, while deflecting the high-temperature stream away from the wall and thus effectively reducing the convective heating of that wall, is itself nonradiative. Such a situation occurs also when a radiation absorbing fluid is injected at a high rate [1] so as to shield wall surfaces against high-density radiation fluxes in plants operating under extreme thermodynamic conditions.

Heat radiation to a permeable surface effects a change in one basic aspect of internal cooling, namely the temperature field of the wall on the exit side. Unlike in the case of porous injection for removal of convective heat, however, in this case the temperature of the porous wall on the exit side may become higher than the mean temperature of the coolant leaving that wall, which can be easily explained on the basis of general physical concepts [2]. The magnitude of this temperature excess, whose maximum allowable magnitude is limited by the thermal stability of the material, represents the critical factor in the design of thermal protection.

In the technical literature this problem has not received as much attention as it merits, which is indicated by the almost complete lack of reports on studies concerning the characteristics of internal heat transfer during the thermal irradiation of a wall. Closest to the subject on hand are the reports in [3, 12] on the internal cooling of a permeable system consisting of parallel circular-section channels (perforation cooling). In [3] the solution to the internal heat transfer problem has been used in deriving an expression for the temperature excess at a permeable plate of infinite thickness. In [12] the authors have analyzed the effect of heat transfer on the entrance side to a wall of finite thickness and the effect of active heat sources generated in the coolant as well as in the wall material by radiation absorbed in the channels, they have also obtained test data on the effectiveness of removing the radiant heat by perforation cooling, and have found these data to agree with analytical results. In [3] and in [12] the problem was formulated on the assumption of a thermally stable flow through the channels and, therefore, on a characterization of the internal heat transfer by a constant heat transfer coefficient* independent of the coolant injection rate. Such an assumption forbids us to extend the results of these calculations to the case of porous cooling, where, as is well known, there exists a very definite relation between the rate of internal heat removal and the injection flow rate of coolant.

* In [12] the Nusselt number was assumed close to its value for steady heat transfer in a pipe at a constant wall temperature, namely Nu = 4; the parabolic profiles of coolant temperature and velocity across a channel section, which were assumed in [3], corresponded to Nu = 6. We note that under the conditions of our coupled internal heat transfer problem the Nusselt number must be expected to depend on the system parameters.

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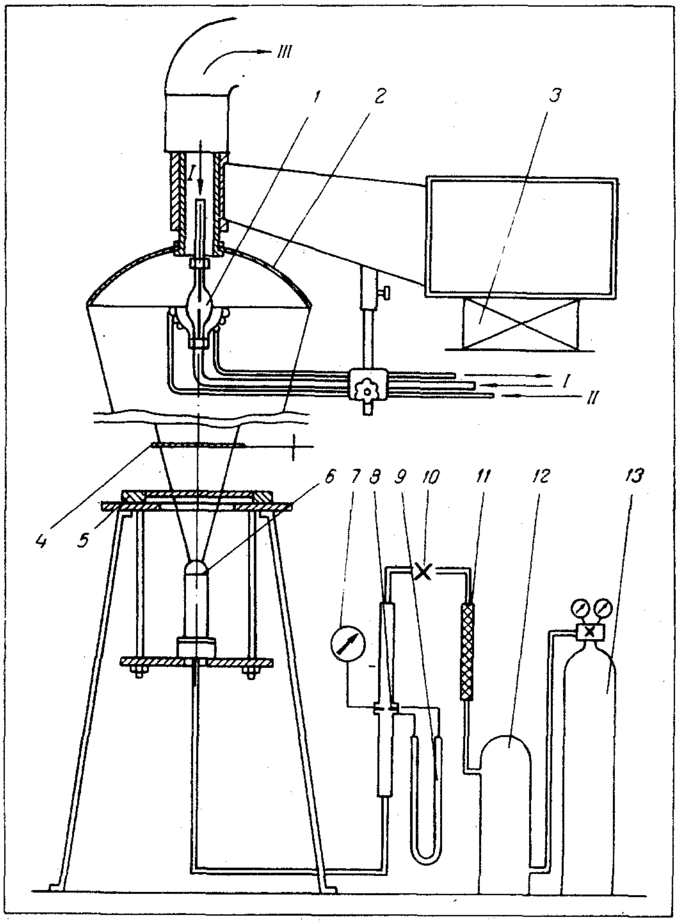

Fig. 1. Test apparatus: 1) model DKSR xenon-filled arc-discharge lamp, 2) ellipsoidal reflector, 3) three-stage coordinate plotter, 4) shutter, 5) grid-type flux attenuator, 6) model, 7) manometer, 8) flow meter diaphragm, 9) differential manometer, 10) flow regulating tap, 11) filter, 12) receiver, 13) high-pressure tank; water (I, II), air for lamp cooling (III).

In connection with the subject matter, we will consider the internal cooling of a plate whose thickness is finite and whose surface is heated mainly by radiation fluxes. The basic equations describing the heat transmission through the wall material and through the coolant fluid respectively can be written as

$$\lambda_w \frac{d^2T_w}{dx^2} = \alpha_v (T_w - T_g), \tag{1}$$

$$-\lambda_w \frac{dT_w}{dx} = \rho v c (T_g - T_{g\infty}). \tag{2}$$

The thermal conductivity of the porous material λ_W corresponds here to a real mechanism of heat transmission through the two-phase system considered here [4] and, therefore, the total energy balance in Eq. (2) does not take into account axial heat conduction through the coolant.

By simple transformations of (1) and (2), one can generalize the energy equation for the porous material [5] in terms of the referred temperature difference $\theta_W = (T_W - T_{g\infty})/(T_{g0} - T_{g\infty})$, which characterizes the sought temperature excess on the exit side of the wall. In dimensionless form, this equation becomes

$$\frac{d^2\theta_w}{d\xi^2} - \frac{\mathrm{Nu}}{\mathrm{Pe}} \cdot \frac{l}{d} \cdot \frac{d\theta_w}{d\xi} - \mathrm{Nu}\frac{\lambda_g}{\lambda_w}\left(\frac{l}{d}\right)^2 \theta_w = 0. \tag{3}$$

The boundary conditions for Eq. (3) defining a one-dimensional temperature field of the wall are stipulated in terms of the thermal flux density at the coolant inlet surface and at the coolant outlet surface.

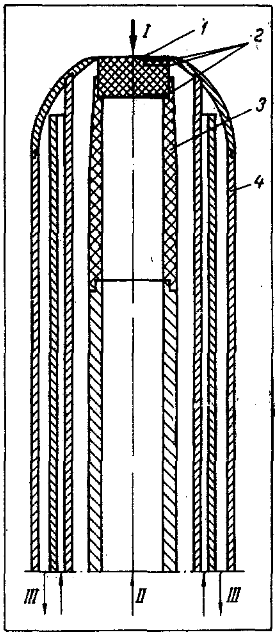

Fig. 2

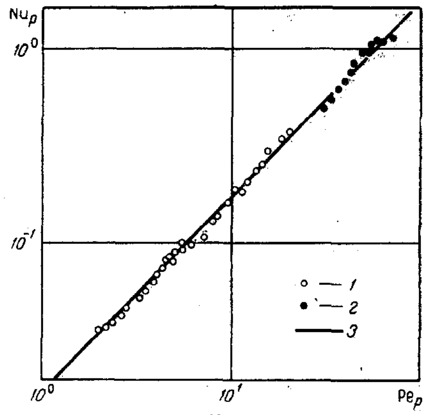

Fig. 3

Fig. 2. Experimental model of a permeable plate: 1) porous insert; 2) thermocouples; 3) thermally insulating sleeve; 4) water-cooled housing; radiation flux (I), injected coolant (II), housing cooling (III).

Fig. 3. Criterial relation describing the internal heat transfer in a porous material: test data with helium injection (1), with argon injection (2), and according to Eq. (13) (3).

The heat received by the wall at the outside surface x = 0 is conducted deeper into the material, according to the relation

$$-\lambda_w \frac{dT_w}{dx} = q_R - \alpha_0 (T_{w0} - T_{g0}). \tag{4}$$

This expression includes the heat transfer from the porous material to the coolant, the temperature of the latter being lower at the exit from the pores [3]. The heat transfer coefficient α_0 referred to the total wall surface area we will define in a form most convenient for further analysis, namely as $\alpha_0 = \lambda_g/\delta$ with δ denoting the equivalent thickness of the thermal boundary layer.

During injection through the wall, the heat content in the coolant increases by the amount of heat supplied by the thermal flux:

$$q_R = \rho vc (T_{g0} - T_{g\infty}). \tag{5}$$

Inserting (5) into Eq. (4) and transforming the latter into dimensionless form yields

$$\xi = 0, \quad \frac{d\theta_w}{d\xi} = -\text{Pe}\frac{\lambda_g}{\lambda_w} \cdot \frac{l}{d} + \frac{\lambda_g}{\lambda_w} \cdot \frac{d}{\delta} \cdot \frac{l}{d} (\theta_{w0} - 1). \tag{6}$$

The temperature gradient in the porous material at x = l corresponds to the heat removed from the wall on the entrance side by coolant injection. In dimensionless form this can be written as

$$\xi = 1, \quad \frac{d\theta_w}{d\xi} = -\text{Nu}' \frac{\lambda_g}{\lambda_w} \cdot \frac{l}{d} \theta_w, \tag{7}$$

where Nu' = $\alpha' d/\lambda_g$.

The solution to Eq. (3) with the boundary conditions (6) and (7) is

$$\theta_w = \left(\frac{\Phi}{2} + \psi\right)\left[\left\{\gamma_2 \frac{\gamma_1 + \omega}{\gamma_2 + \omega} \exp\left[\varphi(\gamma_1 - 1)\right] - \gamma_1\right\}^{-1} \exp\left(\frac{\varphi}{2}\gamma_1\xi\right)\right.$$

$$\left. + \left\{\gamma_1 \frac{\gamma_2 + \omega}{\gamma_1 + \omega} \exp\left[\varphi(\gamma_2 - 1)\right] - \gamma_2\right\}^{-1} \exp\left(\frac{\varphi}{2}\gamma_2\xi\right)\right] \Big/ \Big[1$$

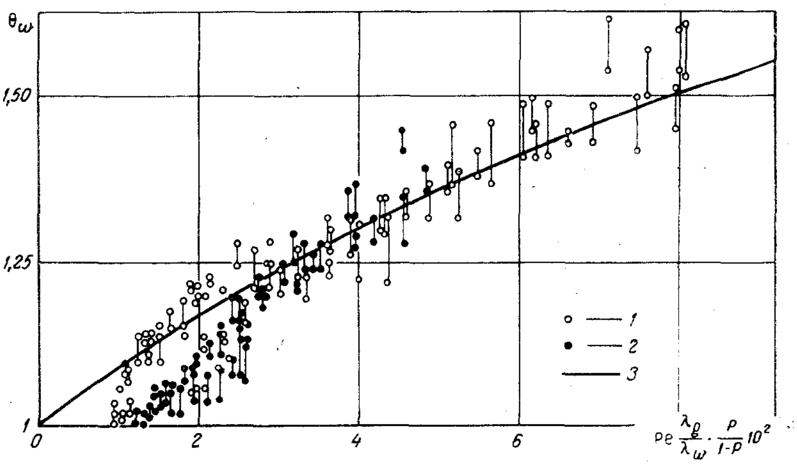

Fig. 4. Temperature excess on the exit side of a porous wall; test data with helium injection (1), with argon injection (2), and according to relation (12) (3).

$$+\psi\left\{\left[\gamma_2\frac{\gamma_1+\omega}{\gamma_2+\omega}\exp\left[\varphi(\gamma_1-1)\right]-\gamma_1\right]^{-1}+\left[\gamma_1\frac{\gamma_2+\omega}{\gamma_1+\omega}\exp\left[\varphi(\gamma_2-1)\right]-\gamma_2\right]^{-1}\right\}\Bigg]. \qquad (8)$$

Here

$$\Phi=\frac{4\mathrm{Pe}^2}{\mathrm{Nu}}\cdot\frac{\lambda_g}{\lambda_w};\qquad \psi=\frac{2\mathrm{Pe}}{\mathrm{Nu}}\cdot\frac{\lambda_g}{\lambda_w}\cdot\frac{d}{\delta};\qquad \varphi=\frac{\mathrm{Nu}}{\mathrm{Pe}}\cdot\frac{l}{d};$$
$$\omega=2\mathrm{Pe}\frac{\mathrm{Nu}'}{\mathrm{Nu}}\cdot\frac{\lambda_g}{\lambda_w};\qquad \gamma_1=1+\sqrt{1+\Phi};\qquad \gamma_2=1-\sqrt{1+\Phi}. \qquad (9)$$

Letting x = 0 in (8), we obtain an expression for the temperature excess on the exit side of the porous wall and this expression can be further simplified by satisfying the additional condition that

$$\varphi(\gamma_1-1)\gg C,\quad \text{where}\quad C\approx 4. \qquad (10)$$

In this case the temperature excess is accurately enough described as a simple function of Φ and ψ:

$$\theta_{w0}=\frac{\Phi/2+\psi}{\sqrt{1+\Phi}-1+\psi}. \qquad (11)$$

The left-hand side of (10) may be regarded as the referred thickness of the porous wall, which, when becoming sufficiently large, will make the temperature of the outside surface independent of the heat transfer rate on the entrance side. It is not difficult to ascertain that condition (10) will always be satisfied during porous cooling, by virtue of the small size of a porous material cell $d/l \ll 1$.

In expression (11) for the temperature difference there appears the thickness of the thermal boundary layer at the exit from the pores, which is determined by all aspects of the flow pattern in the immediate vicinity of the wall and, above all, by the injection rate. It is obviously impossible, within the scope of this study here, to make any suggestions as to how that parameter could be evaluated. An introduction of the ratio $d/\delta \sim 1$ in [3] would not seem based on sufficiently valid reasons. It is logical, however, to assume that at high values of the injection parameters, when the outer stream is almost entirely displaced from the wall and the normal component of velocity becomes predominant at the wall, the rate of external heat removal will not be high enough to govern the temperature field of the porous wall. In this case, if we exclude from consideration the external heat transfer from wall to fluid leaving a pore, the magnitude of the temperature excess found by the proposed solution of the internal heat transfer problem will be somewhat too high and will be the maximum possible under given conditions, which is entirely satisfactory in an evaluation of the thermal stability of structures cooled by porous injection. It is to be noted that an analogous situation exists in practical problems concerning the injection of fluid through a porous wall immersed in a radiating medium.

The temperature excess due to radiative heating of a porous material is defined in terms of the dimensionless group Φ as follows:

$$\theta_{w0}=\frac{\Phi/2}{\sqrt{1+\Phi}-1}. \qquad (12)$$

For the practical use of these relations, it is necessary to know the mode of heat transfer from wall to fluid in the pores. The few and scattered data available today on internal heat transfer in porous media cannot be summarized by a single criterial relation valid for all the diverse coolants and wall materials in modern use, for different porosity levels and injection flow rates. Consequently, the choice of the relation Nu = f(Re, Pr, P) for engineering estimates of the temperature excess in a porous wall according to the proposed formulas seems valid enough.

In order to confirm the results of the preceding theoretical analysis based on certain assumptions, it is necessary to study the effect of the temperature excess in a real situation in an appropriately designed experiment. Therefore, internal cooling of a porous wall under conditions of predominantly radiative heating was studied experimentally on a model in the form of a porous plate thermally insulated around its lateral surface and placed in the focal zone of a high-temperature optical oven with an arc discharge in xenon as the radiation source (Fig. 1) [1].

The porous insert in the model (Fig. 2) was a disc, 9 mm in diameter and 4.75 mm thick, made of stainless steel by the powder metal process of molding and then sintering quasispherical grains d ~ 0.3 mm in diameter. The overall porosity of the material was P = 0.333.

The temperature of the model was measured with copper–constantan thermocouples at various sections of the insert, as shown in Fig. 2. The thermocouple leads were run along isothermal surfaces. The thermal emf was recorded by a model R2/1 semiautomatic potentiometer.

The mean gas temperature on the exit side was calculated from the enthalpy relation:

$$T_{g0} = T_{g\infty} + Q_w/G_g C_g,$$

with the quantity of heat Q_W received by the model found by the transient method in a solid-state calorimeter:

$$Q_w = m_w c_w \frac{dT_w}{d\tau}.$$

The porous insert itself served here as the calorimetering body, whose weight was measured on model ADV-200 analytical scales, while the heating rate was recorded by a model N-700 loop oscillograph. The temperature of the gas before entering the wall was measured with a thermocouple.

The injected gases in this experiment were helium and argon, their thermal conductivities differing substantially. The gas flow rate was determined from the pressure drop across an interchangeable diaphragm which had been precalibrated by the gas meter method. The gas injection rate was varied over a wide range corresponding to a Peclet number Pe ~ 2–20 for helium and Pe ~ 20–100 for argon.

For determining the thermal conductivity of the porous material, the authors used the quasisteady heating method shown in [6] and, accordingly, calculated λ_W from the known value of thermal flux coming to the plate and the temperature difference measurement between two points on that plate during heating:

$$\lambda_w = \frac{0,5q\,(x_2^2 - x_1^2)}{l\,[T_w(x_2) - T_w(x_1)]}.$$

Preliminary qualifying tests were performed, in order to determine the thermal conductivity of a standard specimen made of Armco iron. The result of these tests yielded the same value for λ_W as that given in Tables, which confirmed the suitability of this apparatus for measuring the thermal conductivity of solids. The temperature differences in the tests was recorded on a model N-700 oscillograph by alternative switching of signals from the two respective thermocouples. The coolant (helium or argon) filled the active pore space during these measurements. In this way, we obtained values for the thermal conductivity $\lambda_{W,Ar}$ = 5.8 W/m · deg and $\lambda_{W,He}$ = 5.92 W/m · deg close to one another and, apparently, indicating that heat conduction through contact between particles was predominant in the overall mechanism of heat transfer in the porous wall.

In order to evaluate the test data in terms of the temperature excess on the exit side as a function of the theoretical parameter $\Phi = 4Pe^2\lambda_g/Nu\lambda_W$, which describes the internal cooling process, and then to compare theory with experiment, it is necessary to express this governing parameter in a definitive form based (as has been pointed out earlier) on the rate of internal heat removal Nu as a function of Re, Pr, and P for any specific case. In our study the coefficient of internal heat transfer was determined from the logarithmic mean temperature difference, which in turn was calculated from the temperature drops between wall and fluid on the entrance side and on the exit side [7, 8, 9]. According to the results of these measurements (Fig. 3), the internal cooling mode can be adequately well described by the following empirical relation:

(13)

arrived at by the method of least squares. The form of Eq. (13) agrees with that given by many other authors [7, 9-11] who have also shown the relation Nu = f(Re) to be almost linear. We note that this evaluation of our data takes into account the results in [11], namely that the Prandtl number, which represents the properties of a fluid, is as significant as the Reynolds number in a criterial relation like (13). Relation (13) makes it possible to express the parameter of internal cooling as

$$\Phi = 38.1\mathrm{Pe}\,\frac{\lambda_g}{\lambda_w}\cdot\frac{P}{1-P}\,. \tag{14}$$

In Fig. 4 we compare the temperature excess calculated by formulas (12) and (13) with the corresponding test data.* Evidently, the calculated values do correctly reflect the thermal state of the porous material on the exit side under conditions of predominantly radiative heating. One can now prove, on the basis of relation (12) resulting from this analysis, that a substantial temperature difference between porous material and cooling fluid (over 10%) will be noted at sufficiently high injection flow rates necessary for removing large quantities of heat. It is most worthwhile here to use high-permeability porous materials with a cell size in fractions of a millimeter. For P ~ 0.5-0.3 and for a ratio of thermal conductivities λ_g/λ_w ~0.1-0.01, most typical of light-weight coolant (hydrogen, helium) injection, the Peclet number corresponding to the case considered here ranges approximately from 0.1 to 1. Under moderate or low injection rates through a wall with the pore size in the micron range, the rate of internal heat transfer on the exit side will be so high that the temperature of the porous material there should not be in any significant excess over the temperature of the injected fluid.

Thus, both analysis and experiment have established that, when predominantly radiative heat is removed by porous injection, there arises the problem of internal cooling, the effectiveness of which can be measured in terms of the difference between the temperature of the outside wall surface and the mean fluid temperature on the exit side.

Under high injection rates the maximum temperature of the porous material may be much higher than the temperature of the coolant leaving the wall, and this must be taken into consideration in establishing the operational mode of the given apparatus.

For calculating the temperature excess in the general case of both radiative and convective heat at a porous wall, one needs a physical model describing the heat transfer mechanism in the boundary layer at the exit from pores and one needs appropriate test data.

NOTATION

T	is the temperature;
x	is the transverse coordinate in the wall;
τ	is the time;
q	is the thermal flux density;
Q	is the thermal flux;
G	is the flow rate;
m	is the mass;
ρ	is the density;
c	is the specific heat at constant pressure;
λ	is the thermal conductivity;
v	is the velocity;
α_V	is the coefficient of volume heat transfer;
α'	is the coefficient of heat transfer to the wall on the entrance side;
δ	is the thickness of thermal boundary layer;
d	is the particle diameter;
l	is the thickness of wall;
P	is the porosity;
$\mathrm{Pe} = \rho vcd/\lambda_g$	is the Peclet number;

* The scatter of test values for θ_{w0} is attributable to the thermal instability of the radiation condenser.

$Nu = \alpha_V d^2 / \lambda_g$	is the Nusselt number;
Pr	is the Prandtl number;
$Nu_p = Nu/6(1-P)$	is the Nusselt number for flow through a porous medium;
$Pe_p = Pe/P$	is the Peclet number for flow through a porous medium;
$\theta_W = (T_W - T_{g\infty})/(T_{g0} - T_{g\infty})$	is the dimensionless temperature;
$\xi = x/l$	is the dimensionless transverse coordinate.

Subscripts

0 denotes to the outside wall surface;
∞ denotes to the region before the entrance of fluid into the wall;
w denotes to the wall;
g denotes to the coolant fluid (gas);
p denotes to the porous medium.

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